Laser Interferometer Tracking System Signal Processing

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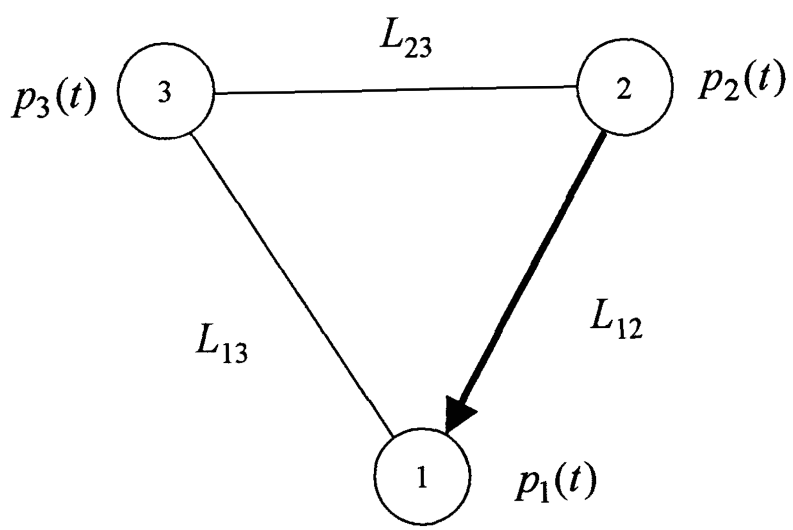
ABSTRACT:
Algorithms for processing the LISA Laser Interferometer Tracking System (LITS) data are presented. We begin with a review of the time-domain algorithms for eliminating laser phase noise previously reported by Armstrong, Estabrook, and Tinto (AET, 1999). We then present a new time-domain set of algorithms for eliminating noise due to jitter in the spacecraft frequency standards, one set for each of the AET algorithms. Details of the signal acquisition for each of these cases are presented and the resulting data rate requirements are analyzed. Finally, we summarize the requirements in each of two implementations of the laser locking scheme: 1) phase-locking the lasers on the three spacecraft together, and 2) independent cavity-locked lasers in each spacecraft. We also present an analysis of the two suggested implementations of the two-color laser system: 1) two lasers locked to the same cavity, and 2) phase modulation of a single laser.

Spacecraft Configuration



$y_{21}(t) =$ signal from S/C 2 received at S/C 1 at time t

$$y_{21}(t) = p_2(t - L_{12}) - p_1(t) + \text{signals} + \text{noise}$$

Signals Free of Laser Phase Noise

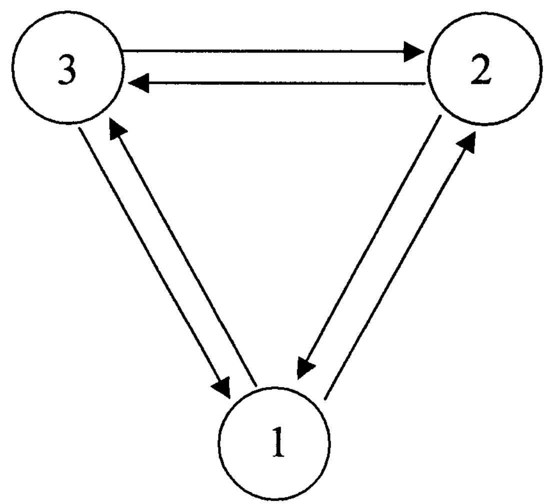
(see Armstrong, Estabrook, & Tinto)

- A little careful algebra shows that one may create combinations of signals that are free of laser phase noise

$$A(t) = y_{31}(t) - y_{21}(t) + y_{23}(t - L_{13}) - y_{32}(t - L_{12}) + y_{12}(t - L_{13} - L_{23}) - y_{13}(t - L_{12} - L_{23})$$

$B(t) = [1 \rightarrow 2 \rightarrow 3]$

$C(t) = [1 \rightarrow 2 \rightarrow 3 \text{ again}]$

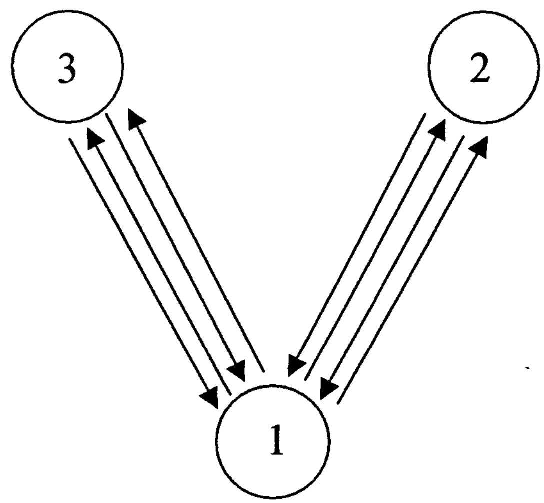


Interferometer-like Signals

$$X(t) = y_{12}(t - L_{12} - 2L_{13}) - y_{13}(t - L_{13} - 2L_{12}) + y_{21}(t - 2L_{13}) - y_{31}(t - 2L_{12})$$
$$+ y_{13}(t - L_{13}) - y_{12}(t - L_{12}) + y_{31}(t) - y_{21}(t)$$

$$Y(t) = [1 \rightarrow 2 \rightarrow 3]$$

$$Z(t) = [1 \rightarrow 2 \rightarrow 3 \text{ again}]$$



Lock to the Incoming Signal or Not?

$$X(t) = y_{12}(t - L_{12} - 2L_{13}) - y_{13}(t - L_{13} - 2L_{12}) + y_{21}(t - 2L_{13}) - y_{31}(t - 2L_{12})$$
$$+ y_{13}(t - L_{13}) - y_{12}(t - L_{12}) + y_{31}(t) - y_{21}(t)$$

Lock to the Incoming Signal or Not?

$$X(t) = y_{12}(t - L_{12} - 2L_{13}) - y_{13}(t - L_{13} - 2L_{12}) + y_{21}(t - 2L_{13}) - y_{31}(t - 2L_{12})$$
$$+ y_{13}(t - L_{13}) - y_{12}(t - L_{12}) + y_{31}(t) - y_{21}(t)$$

- With signals locked so that $y_{12}(t) = 0$ and $y_{13}(t) = 0$, $X(t)$ simplifies to

$$X(t) = y_{21}(t - 2L_{13}) - y_{31}(t - 2L_{12}) + y_{31}(t) - y_{21}(t)$$

$\boxed{\Rightarrow \text{1 channel only}}$

Lock to the Incoming Signal or Not?

$$X(t) = y_{12}(t - L_{12} - 2L_{13}) - y_{13}(t - L_{13} - 2L_{12}) + y_{21}(t - 2L_{13}) - y_{31}(t - 2L_{12})$$
$$+ y_{13}(t - L_{13}) - y_{12}(t - L_{12}) + y_{31}(t) - y_{21}(t)$$

$$Y(t) = y_{23}(t - L_{23} - 2L_{12}) - y_{21}(t - L_{12} - 2L_{23}) + y_{32}(t - 2L_{12}) - y_{12}(t - 2L_{23})$$
$$+ y_{21}(t - L_{12}) - y_{23}(t - L_{23}) + y_{12}(t) - y_{32}(t)$$

- With signals locked so that $y_{12}(t) = 0$ and $y_{13}(t) = 0$, simplifies to

$$X(t) = y_{21}(t - 2L_{13}) - y_{31}(t - 2L_{12}) + y_{31}(t) - y_{21}(t)$$

$\Rightarrow$ 1 channel only

$$Y(t) = y_{23}(t - L_{23} - 2L_{12}) - y_{21}(t - L_{12} - 2L_{23}) + y_{32}(t - 2L_{12})$$
$$+ y_{21}(t - L_{12}) - y_{23}(t - L_{23}) - y_{32}(t)$$

$\Rightarrow$ y_{23}, y_{32}, and y_{21} are also needed

How Does It Work?

- The format of each received signal is

$$y_{ij} = p_i(t - L_{ij}) - p_j(t) + \underbrace{h_{ij}(t)}_{\text{signal}} + \overbrace{n_{ij}(t)}^{\text{other noise}}$$

- So

$$\begin{aligned} X(t) = {} & y_{12}(t - L_{12} - 2L_{13}) - y_{13}(t - L_{13} - 2L_{12}) + y_{21}(t - 2L_{13}) - y_{31}(t - 2L_{12}) \\ & + y_{13}(t - L_{13}) - y_{12}(t - L_{12}) + y_{31}(t) - y_{21}(t) \end{aligned}$$

becomes

$$\begin{aligned} X(t) = {} & p_1(t - 2L_{12} - 2L_{13}) - p_2(t - L_{12} - 2L_{13}) + h_{12}(t - L_{12} - 2L_{13}) + n_{12}(t - L_{12} - 2L_{13}) \\ & - p_1(t - 2L_{13} - 2L_{12}) + p_3(t - L_{13} - 2L_{12}) - h_{13}(t - L_{13} - 2L_{12}) - n_{13}(t - L_{13} - 2L_{12}) \\ & + p_2(t - 2L_{13} - L_{12}) - p_1(t - 2L_{13}) + h_{12}(t - 2L_{13}) + n_{21}(t - 2L_{13}) \\ & - p_3(t - 2L_{12} - L_{13}) + p_1(t - 2L_{12}) - h_{13}(t - 2L_{12}) - n_{31}(t - 2L_{12}) \\ & + p_1(t - 2L_{13}) - p_3(t - L_{13}) + h_{13}(t - L_{13}) + n_{13}(t - L_{13}) \\ & - p_1(t - 2L_{12}) + p_2(t - L_{12}) - h_{12}(t - L_{12}) - n_{12}(t - L_{12}) \\ & + p_3(t - L_{13}) - p_1(t) + h_{13}(t) + n_{31}(t) \\ & - p_2(t - L_{12}) + p_1(t) - h_{12}(t) - n_{21}(t) \end{aligned}$$

How Does It Work? – cont'd

$$X_n = \frac{1}{\Delta t} \int_{t_n}^{t_n+\Delta t} X(t)\, dt$$

$$= \frac{1}{\Delta t} \int_{t_n}^{t_n+\Delta t} y_{12}(t - L_{12} - 2L_{13})\, dt \; - \int_{t_n}^{t_n+\Delta t} y_{13}(t - L_{13} - 2L_{12}) dt + \int_{t_n}^{t_n+\Delta t} y_{21}(t - 2L_{13}) dt$$

$$- \int_{t_n}^{t_n+\Delta t} y_{31}(t - 2L_{12}) dt + \int_{t_n}^{t_n+\Delta t} y_{13}(t - L_{13}) dt - \int_{t_n}^{t_n+\Delta t} y_{12}(t - L_{12}) dt + \int_{t_n}^{t_n+\Delta t} y_{31}(t) dt - \int_{t_n}^{t_n+\Delta t} y_{21}(t) dt$$

signals	times			
y_{12}	$t - L_{13} - 2L_{12}$	$t - L_{12}$		
y_{13}	$t - L_{13} - 2L_{12}$	$t - L_{13}$		
y_{21}			$t - 2L_{13}$	t
y_{31}			$t - 2L_{12}$	t

How Does It Work? – cont'd

For all three signals: $X(t)$, $Y(t)$, and $Z(t)$ one must have:

signals	times			
y_{21}	t-L_{12}-$2L_{23}$	t-L_{12}	t-$2L_{13}$	t
y_{31}	t-L_{13}-$2L_{23}$	t-L_{13}	t-$2L_{12}$	t
y_{12}	t-L_{12}-$2L_{13}$	t-L_{12}	t-$2L_{23}$	t
y_{32}	t-L_{23}-$2L_{13}$	t-L_{23}	t-$2L_{12}$	t
y_{13}	t-L_{13}-$2L_{12}$	t-L_{13}	t-$2L_{23}$	t
y_{23}	t-L_{23}-$2L_{12}$	t-L_{23}	t-$2L_{13}$	t

$\Rightarrow$ not needed for $X(t)$ and $Y(t)$ only

Tentative Conclusions

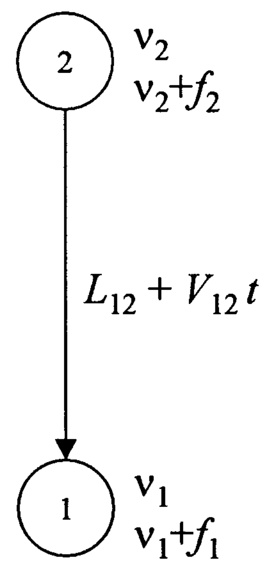
1. Minimum number of channels: $X(t)$ and $Y(t)$
   - locked lasers $\Rightarrow$ 4 signals
   - independent lasers $\Rightarrow$ 6 signals

2. All three channels: $X(t)$, $Y(t)$, and $Z(t)$
   - locked lasers $\Rightarrow$ 7 signals
   - independent lasers $\Rightarrow$ 9 signals

- Read out all one-way signals
- Independent lasers in each spacecraft, locked to their own cavities
- Each signal readout system must simultaneously track and record 4 signals, each with a particular time offset relative to the nominal sample time

Frequency Standard Correction Algorithms

(The Two-Color Laser System)



S/C #2 emits two laser phases that are received by S/C #1:

$$\phi_2(t) = \nu_2\, t\,(1 - V_{12}) + p_2\,(t - L_{12})$$

$$\phi'_2(t) = (\nu_2 + f_2)\; t\; (1 - V_{12}) + p_2(t - L_{12}) + q_2\,(t - L_{12})$$

S/C #1 generates two laser phases:

$$\phi_1(t) = \nu_1\, t + p_1\,(t)$$

$$\phi'_1(t) = (\nu_1 + f_1)\, t + p_1(t) + q_1\,(t)$$

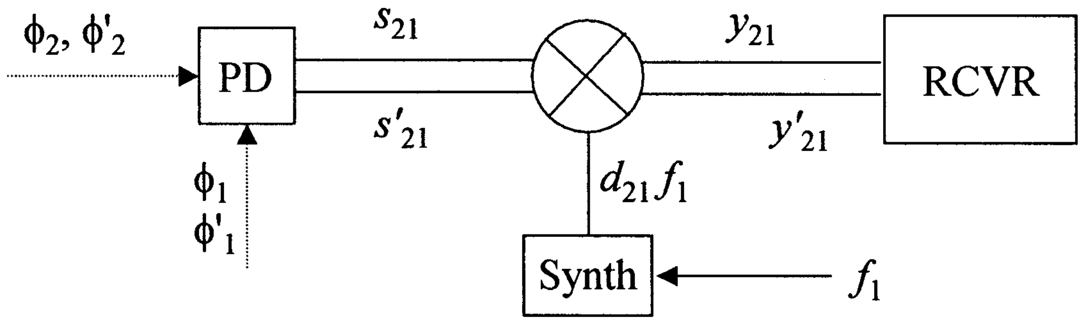
S/C #1 beats its local laser against the incoming laser to create two signals:

$$s_{21} = [(\nu_2 - \nu_1) - V_{12}\nu_2]t + p_2(t - L_{12}) - p_1(t)$$

$$s'_{21} = [(\nu_2 - \nu_1) + (f_2 - f_1) - V_{12}(\nu_2 + f_2)]t + p_2(t - L_{12}) - p_1(t) + q_2(t - L_{12}) - q_1(t)$$

Frequency Standard Correction Algorithms – cont'd

- These signals are at frequencies close to $\nu_2 - \nu_1$. A local oscillator is set at frequency $d_{21} f_1$ in order to beat this frequency down to a baseband where it may be sampled.



$$y_{21}(t) = s_{21}(t) - d_{21}[f_1 t + q_1(t)] = [(\nu_2 - \nu_1) - V_{12}\nu_2 - d_{21} f_1]t + p_2(t - L_{12}) - p_1(t) - d_{21} q_1(t)$$

$$y'_{21}(t) = s'_{21}(t) - d_{21}[f_1 t + q_1(t)] = [(\nu_2 - \nu_1) + (f_2 - f_1) - V_{12}(\nu_2 + f_2) - d_{21} f_1]t$$
$$+ p_2(t - L_{12}) - p_1(t) + q_2(t - L_{12}) - (d_{21} + 1) q_1(t)$$

$$r_{21}(t) = y'_{21}(t) - y_{21}(t) = [(f_2 - f_1) - V_{12}\overset{f_2}{\cancel{\nu_2}}]t + q_2(t - L_{12}) - q_1(t)$$

Frequency Standard Correction Algorithms – cont'd

- Neglecting the constant rate ν_i and the quasi-constant $V_{ij}\,\nu_i\,t$ terms, we write the signals required for data processing as:

$$y_{ij}(t) = p_i(t - L_{ij}) - d_{ij} q_j(t)$$

$$r_{ij}(t) = q_i(t - L_{ij}) - q_j(t)$$

- When the $X(t)$ signal is formed, the residual noise terms are:

$$X(t) = d_{12}[q_2(t - L_{12} - 2L_{13}) - q_2(t - L_{12})] - d_{13}[q_3(t - L_{13} - 2L_{12}) - q_3(t - L_{13})]$$
$$+ d_{21}[q_1(t - 2L_{13}) - q_1(t)] - d_{31}[q_1(t - L_{12}) - q_1(t)]$$

- However, a little algebra shows that the combination of $X(t)$ and $r_{ij}(t)$ given by

$$\xi(t) = X(t) - d_{12} r_{21}(t - 2L_{13}) + d_{13} r_{31}(t - 2L_{12}) - (d_{12} - d_{21}) r_{13}(t - L_{13})$$
$$+ (d_{13} + d_{31}) r_{12}(t - L_{12}) + (d_{12} + d_{13} + d_{31}) r_{21}(t) - (d_{13} + d_{12} + d_{21}) r_{31}(t)$$

will be completely free of both laser phase noise and frequency jitter noise.

Frequency Standard Correction Algorithms – cont'd

- The complete set of noise-free signals for $X(t)$, $Y(t)$ and $Z(t)$ is given by

$$\xi(t) = X(t) - d_{12}r_{21}(t-2L_{13}) + d_{13}r_{31}(t-2L_{12}) - (d_{12}+d_{21})r_{13}(t-L_{13}) \\ + (d_{13}+d_{31})r_{12}(t-L_{12}) + (d_{12}+d_{13}+d_{31})r_{21}(t) - (d_{13}+d_{12}+d_{21})r_{31}(t)$$

$$\psi(t) = Y(t) - d_{23}r_{32}(t-2L_{12}) + d_{21}r_{12}(t-2L_{23}) - (d_{23}+d_{32})r_{21}(t-L_{12}) \\ + (d_{21}+d_{12})r_{23}(t-L_{23}) + (d_{23}+d_{21}+d_{12})r_{32}(t) - (d_{21}+d_{23}+d_{32})r_{12}(t)$$

$$\zeta(t) = Z(t) - d_{31}r_{13}(t-2L_{23}) + d_{32}r_{23}(t-2L_{13}) - (d_{31}+d_{13})r_{32}(t-L_{23}) \\ + (d_{32}+d_{23})r_{31}(t-L_{13}) + (d_{31}+d_{32}+d_{23})r_{13}(t) - (d_{32}+d_{31}+d_{13})r_{23}(t)$$

- An approximate solution for the $A(t)$, $B(t)$, $C(t)$ set of signals has also been found: *e.g.*

$$\alpha(t) = A(t) + (d_{23}-d_{21})r_{23}(t-L_{13}) - d_{21}r_{31}(t) + (d_{31}-d_{32})r_{32}(t-L_{12}) \\ + d_{31}r_{21}(t) + (d_{13}-d_{31}+d_{32})[r_{12}(t-L_{13}-L_{23}) - r_{13}(t-L_{12}-L_{13})]$$

How Does It Work? – cont'd

For all three signals: $X(t)$, $Y(t)$, and $Z(t)$ one must have:

signals	times			
y_{21}	$t-L_{12}-2L_{23}$	$t-L_{12}$	$t-2L_{13}$	t
y_{31}	$t-L_{13}-2L_{23}$	$t-L_{13}$	$t-2L_{12}$	t
y_{12}	$t-L_{12}-2L_{13}$	$t-L_{12}$	$t-2L_{23}$	t
y_{32}	$t-L_{23}-2L_{13}$	$t-L_{23}$	$t-2L_{12}$	t
y_{13}	$t-L_{13}-2L_{12}$	$t-L_{13}$	$t-2L_{23}$	t
y_{23}	$t-L_{23}-2L_{12}$	$t-L_{23}$	$t-2L_{13}$	t

$\Rightarrow$ not needed for $X(t)$ and $Y(t)$ only

Frequency Standard Correction Algorithms – cont'd

- For all three signals, $\xi(t)$, $\psi(t)$, and $\zeta(t)$, one must additionally have:

signals	times		
r_{21}	t-L_{12}	t-$2L_{13}$	t
r_{31}	t-L_{13}	t-$2L_{12}$	t
r_{12}	t-L_{12}	t-$2L_{23}$	t
r_{32}	t-L_{23}	t-$2L_{12}$	t
r_{13}	t-L_{13}	t-$2L_{23}$	t
r_{23}	t-L_{23}	t-$2L_{13}$	t

Conclusions

Case 1. Inter-Spacecraft Communications

- Independent lasers $\Rightarrow$ 9 signals sent by each S/C to the central S/C
- Phase-locked lasers $\Rightarrow$ 8 signals sent by each S/C to the central S/C
- Phase-locked clock $\Rightarrow$ 5 signals sent by each S/C to the central S/C
- On-board signal processing with 3 signals telemetered to earth

Case 2. Telemetry From Each Spacecraft to Earth

- Independent lasers $\Rightarrow$ 9 signals sent by each S/C to Earth
- Phase-locked lasers $\Rightarrow$ 8 signals sent by each far S/C to earth
  plus 2 signals from the central S/C to Earth
- Phase-locked clock $\Rightarrow$ 5 signals sent by each far S/C to Earth
  plus 2 signals from the central S/C to Earth